A Logarithmic Transmission Line Chart*

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Summary—**A chart is presented which relates the real and imaginary components of the impedance at any position along a transmission line to the magnitude and location of the standing wave. In the present chart the ordinate is R/Z_0 plotted logarithmically and the abscissa is a function of X/R. Thus a change in the reference impedance becomes a simple vertical translation of any point. An auxiliary chart permits the direct determination of the length and impedance of transmission line required to match a given impedance.**

Introduction

TRANSMISSION line impedance charts are well known.[1] There are, for example, the rectangular impedance chart and the circular or "Smith" chart,[2] which deal with a mismatched transmission line and relate the magnitude and position of the standing wave to the real and imaginary components of the impedance (or admittance) at any point along the transmission line.

The above description applies also to the present chart (Fig. 1). This chart has been designed for quick solution of problems in which the characteristic impedance of the transmission line varies discontinuously along its length. It is believed to be more convenient than the Smith chart for problems where successive lengths of line of varying impedance are connected in cascade and less convenient for certain other problems; for example, where shunt reactive stubs are connected along the line.

Requirements of an Impedance Chart

To achieve simplicity when the characteristic impedance of the transmission line is subject to discontinuous changes, a chart should be designed so that one of the axes represents some function of the phase angle ϕ, where the impedance Z is written as $|Z|\angle\phi$. The simplest arrangement is to plot ϕ itself on a linear scale. This has been done here; however, the ϕ scale is not shown, rather the abscissa has been calibrated at even values of X/R. X/R is, of course, $\tan^{-1}\phi$; X and R are the series reactance and resistance, respectively. The advantage of using ϕ or some function of it for one rectangular axis is that a change of reference impedance represents a straight-line motion parallel to one axis since the phase angle ϕ is independent of the reference impedance.

The other requirement of such a chart is that the other axis be log $|Z|$, or log R, or log X. The second alternative, log R, has been chosen here. Because of the logarithmic scale, a change of reference impedance represents the same amount of motion at any location on the chart. The choice of log R rather than log $|Z|$, and of X/R rather than ϕ, was made for easy use of data in the $R+jX$ form. This facilitates the interchange of data between the present chart and the Smith chart, for example. The rectangular form is also the most usual when using an impedance bridge. It is recognized that a log $|Z|$, ϕ chart would be more elegant since it just represents the complex Z plane and would also be more useful when it is necessary to express the impedance in the polar form.

Range of the Chart

Because the ordinate is a logarithmic scale, some limitation is necessary. Here, for convenience, R/Z_0 has been limited to the range 0.1 to 10. To be consistent with this, the horizontal range has been chosen to include all points having a voltage standing wave ratio of 10 or less. Angles between 80° and 90° have not been included because in this region the two families of curved lines are too nearly parallel for convenient use. This difficulty would not arise with the log Z chart.

Description and Use of the Chart

To be given the VSWR as a number greater than unity which will select one of the VSWR loci, seen to be roughly concentric with the center of the chart, and to be given the position of the voltage minimum, measured in electrical degrees from some reference point, is the most usual way to enter the chart. This position will select one of the radial lines marked in degrees toward the generator from the point of minimum impedance. Movement towards the generator will represent a counterclockwise motion on the chart.

The abscissa of the chart is drawn (but not calibrated) to represent, on a linear scale, the phase angle of the impedance; that is, $\tan^{-1} X/R$. X and R are the components of the impedance at the point in question. The range is $-80°$ to $+80°$, angles between 80° and 90° being off the chart. The degrees scale is not shown; however, for convenience the abscissa is calibrated to the scale X/R.

It will be noted that X/R is independent of the reference characteristic impedance. Hence a change of impedance in the line will not change the abscissa, or in other words, reference impedance changes will be represented by vertical movements on the chart.

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[1] P. S. Carter, "Charts for transmission-line measurements and computations," *RCA Rev.*, vol. 3, pp. 355–368; January, 1939.

[2] P. H. Smith, "Transmission line calculator," *Electronics*, vol. 12, pp. 29–31; January, 1939.

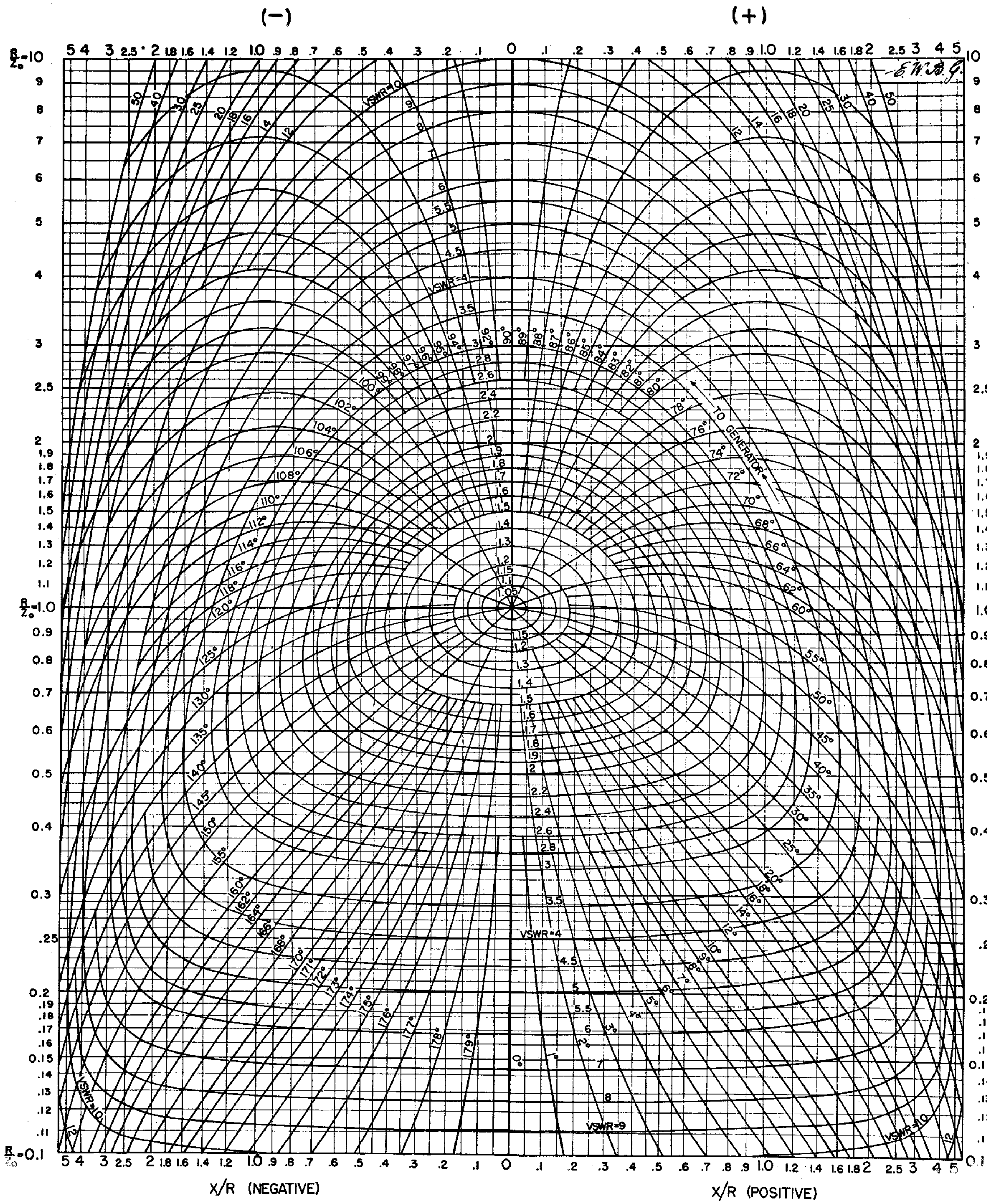


Fig. 1—Logarithmic transmission line chart.

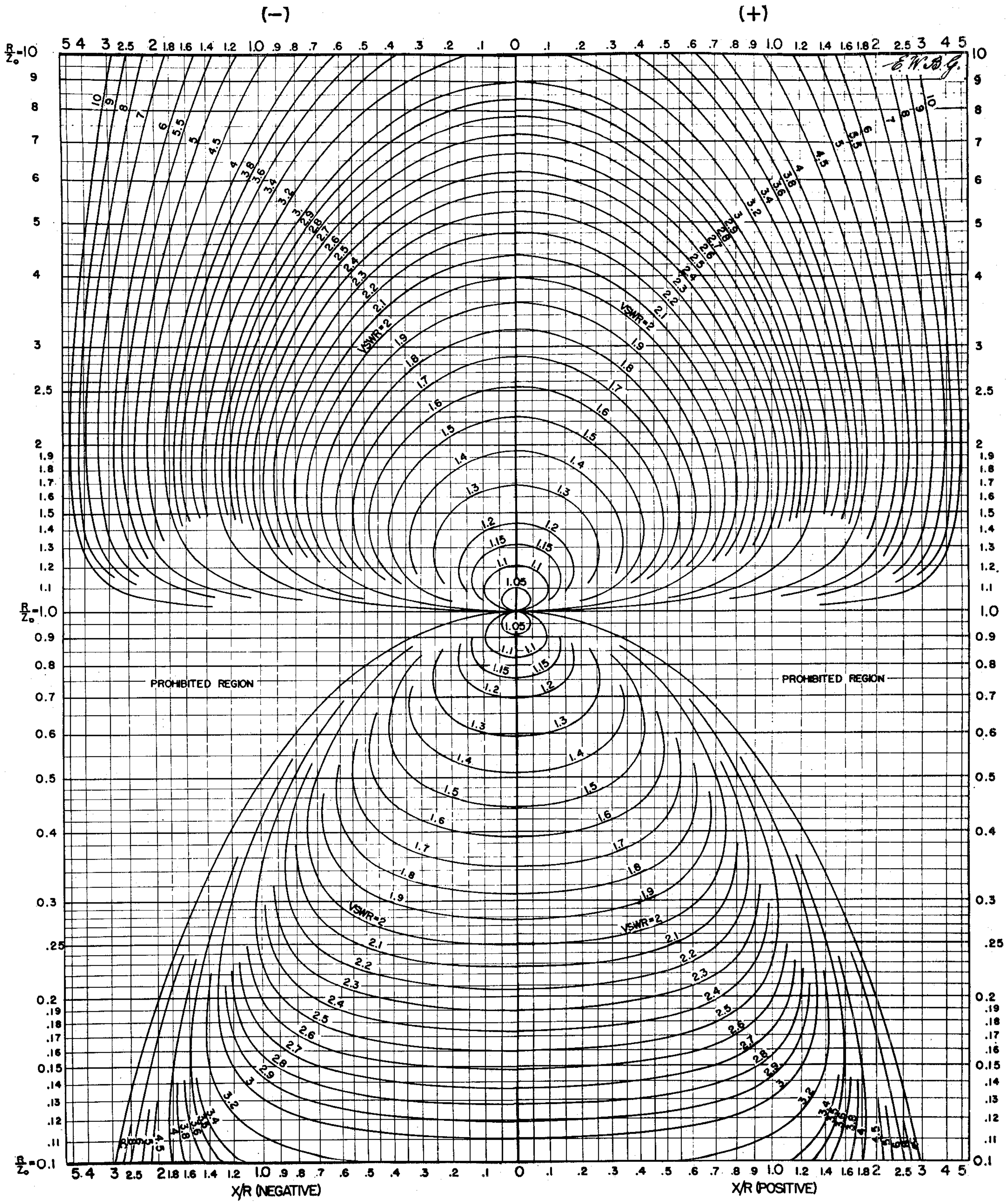


Fig. 2—Transposed logarithmic transmission line chart.

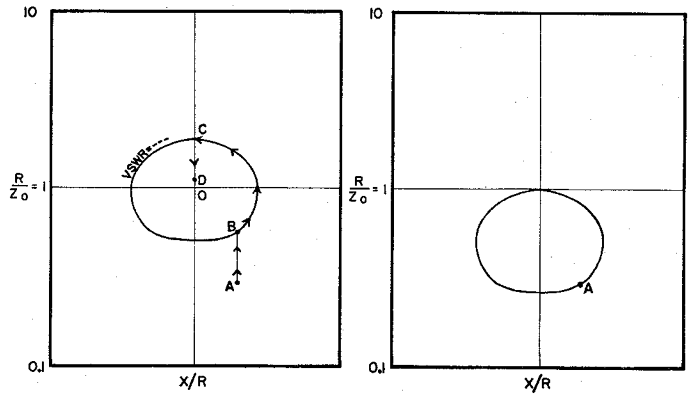
The ordinate of the chart is R/Z_0, where Z_0 is the characteristic impedance of the transmission line. Because of the logarithmic scale, a change in reference impedance of a certain ratio will represent the same amount of vertical movement anywhere on the chart.

Consider a point plotted to a reference impedance of, say, 50 ohms. Assume that we wish to move along a 50-ohm line $\frac{1}{8}$ wavelength toward the generator. We follow the VSWR locus in a counterclockwise direction, counting off 45° on the degrees scale. Now assume that the transmission line changes at this point to one having a characteristic impedance of 75 ohms. R/Z_0 will be reduced to 50/75 of its former value. Thus we move vertically down on the chart, the distance moved being found by setting a pair of dividers on the ordinate scale between any two points which are in the ratio 50:75, for example, 0.5 and 0.75. We may now move along the new standing-wave ratio locus counterclockwise for the number of electrical degrees corresponding to the length of the section of 75-ohm line. To return now to the original 50-ohm reference, the point is moved vertically upwards by the same divider setting.

The use of the chart for a typical broad-band matching problem will be discussed later, but first the transposed chart, Fig. 2, will be described.

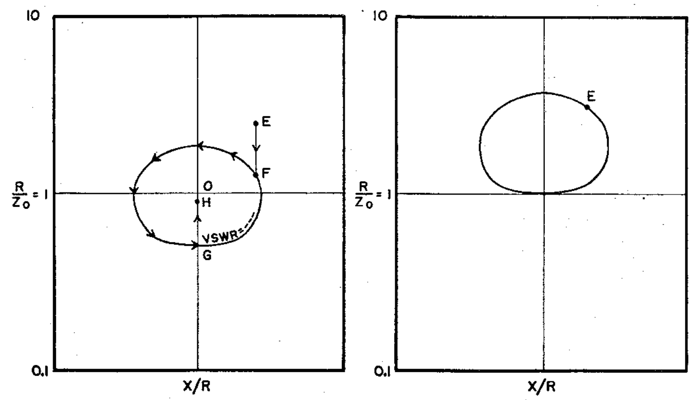
Transposed Chart

The purpose of Fig. 2 is to answer the question, "What length of line of what impedance will match a given impedance?" To put the question in a more practical form, "Of those transmission lines which are available, which impedance is the best, and what length should be used?" Conventional charts do not answer this question explicitly. The present chart (Fig. 1) is somewhat better, but one trial-and-error operation with a pair of dividers is still necessary.



(a) (b)

Fig. 3—(a) Diagrammatic representation of the chart of Fig. 1 with only one VSWR locus shown. If this locus is the correct one to match point A, then point D will coincide with O. (b) Transposed version of (a). Here the locus in (a) has been moved bodily downward. If it is the correct locus, it will pass through A.



(a) (b)

Fig. 4—(a) Diagrammatic representation of the chart of Fig. 1 with only one VSWR locus shown. If this locus is the correct one to match point E, then point H will coincide with O. (b) Transposed version of (a). Here the locus in (a) has been moved bodily upward. If it is the correct locus, it will pass through E.

Suppose for example that we wish to match an impedance represented by point A [Fig. 3(a)] point A being referred to the impedance of the standard transmission line which is in use. Note that Fig. 3(a) is a representation of Fig. 1. We transform from point A to B; in other words, a transmission line which is lower in impedance by the ratio represented by the distance AB is used. We now move counterclockwise along the constant VSWR locus to point C, the point of zero reactance. The length of the lower-impedance line is given by the difference in the degree readings at the points B and C. The transformation back to the standard impedance is represented by the downward move CD. For a perfect match, we wish point D to coincide with O. Thus, in terms of Fig. 1, to match point A (Fig. 3), such a VSWR locus that $AB=CO$ is sought; this is a trial-and-error procedure.

But suppose that the VSWR locus shown on Fig. 3(a) is the correct locus to match point A; in other words, suppose that $AB=CO$. Now suppose also that the VSWR locus is moved bodily downward until it is tangent to the $R/Z_0=1$ axis, as indicated in Fig. 3(b). Distance CO will collapse to zero and since $AB=CO$, AB will collapse to zero also. Thus the transposed locus will pass through point A. Suppose finally that all of the VSWR loci in Fig. 1 are translated downward until they are tangent to the $R/Z_0=1$ axis, then the VSWR locus now passing through any point A is the correct one to use for matching the impedance represented by point A. It will be noted that the impedance transformation ratio for a correct match is always equal to V, the VSWR on the matching section. The lower half of Fig. 2 has been prepared by sliding the VSWR loci in the manner described above.

Now, Fig. 3 deals with an impedance whose normalized resistive component, R/Z_0, is less than unity and requires a lower impedance line for matching. If, on the other hand, the value of R/Z_0 is greater than unity, we refer to Fig. 4. In Fig. 4(a) it is assumed that we wish to match point E. In an exactly analogous manner we move down to point F, then counterclockwise to G, and up to H. Thus by sliding all the VSWR loci upward, as has been done in the upper half of Fig. 2, the correct VSWR may be read directly.

One way of presenting the data of Fig. 2 would be to superimpose it on Fig. 1; however, in the absence of two-color printing, Fig. 1 would become difficult to read. Figs. 1 and 2 are the same size however, so a point from Fig. 1 may be conveniently transferred to Fig. 2 with dividers or compass.

Use of the Transposed Chart

For greater clarity, the use of Fig. 2 will now be described without any references to the method of formation of the chart. Given an impedance normalized to Z_0, and represented by some point on Fig. 1, it is required to find the length and impedance of a section of transmission line which will match the given impedance to a line of characteristic impedance Z_0.

1) Transfer the point geometrically to Fig. 2.

2) Read the value of VSWR indicated on this chart. Let us assume that the value found is V. No more use is made of Fig. 2.

3) Draw a line vertically[3] from the original point on Fig. 1 to intersect the VSWR curve of value V.

4) Note that the impedance transformation ratio represented by the length of this line is exactly V, and in fact, best accuracy in the previous step is obtained by setting a pair of dividers at this ratio, as determined by the ordinate scale.

5) a) If the original point is in the lower half plane, the characteristic impedance of the matching section is Z_0 *decreased* in the above ratio V. b) If the original point is in the upper half plane, the impedance of the matching section is Z_0 *increased* in the ratio V.

6) To find the length of the matching section, move from the intersection on curve V (see step 3) counterclockwise to the first zero reactance point which is in the opposite half plane (vertically) to the original point.

7) The number of degrees passed in this motion gives the length of the matching line.

It may be seen from Fig. 4 that all points in the upper half-plane may be matched, but matching of some impedances which have $R/Z_0<1$ is impossible. For comparison, one may refer to published analytical expressions[4] for the length and impedance of the matching section.

[3] The following rules will define which of the two possible intersections is used: 1) if the original point is in the upper half-plane the motion is downward and vice versa; 2) the first intersection encountered is the correct one.

[4] "Very High-Frequency Techniques," Radio Res. Lab., Harvard Univ., McGraw-Hill Book Co., Inc., New York, N. Y., and London, Eng. (first ed.), vol. 1, p. 60; 1947.

A Typical Broad-Band Matching Problem

This problem will be described very briefly. A model of an antenna for use with a 50-ohm system was measured with a standing-wave line, eight points being measured in the band 96 to 300 mc. (These were model-testing frequencies; the actual antenna was to operate at frequencies lower by a factor of 48.) The points were plotted on a chart similar to Fig. 1, using the VSWR and position of the minimum to locate the points. The resultant curve made several excursions over the chart and reached some values of VSWR as high as 12. The position of the curve, mainly in the upper half-plane, suggested the use of a matching section of higher impedance. So as a first trial a 97-ohm line, having an electrical length of 10° at 100 mc, was tried. This line was thought of as having a length of 1° per 10 mc. Using one divider setting of the ratio 97 to 50, the points were moved down, then around counterclockwise, increasing the angle reading by an appropriate amount for each frequency, for example, a 17.5° increase at 175 mc. Then, with the same divider setting, the points were moved vertically upward, thus giving the final VSWR. No writing was done on the chart itself, one point being treated at a time and the final VSWR being plotted on a linear scale of VSWR vs frequency. One single setting of a slide rule gave the electrical length for all frequencies. The final choice was a 75-ohm line of length 2° per 10 mc, which matched the antenna to within a VSWR of 3.5. This provided a very economical matching device since it was necessary only to replace 17.3 feet of 50-ohm cable with 75-ohm cable (17.3 feet in the actual antenna, not the scale model). This type of problem, where the final match requirement is not severe, is often amenable to this kind of matching. The time required to plot each VSWR vs frequency curve was about six minutes.

Use of the Chart for Admittance

As with the conventional impedance charts, the present chart may be used to represent admittance instead of impedance, the abscissa scale then being B/G, and the ordinate scale G/Y_0. The right-hand (positive) side of the chart which is inductive for the impedance chart becomes capacitive when the chart is used as an admittance chart and vice versa. To change from impedance to admittance the point is moved 90° in either direction along a line of constant VSWR.

Acknowledgment

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